Name of the Papers	: **Complex Analysis (UPC- 32351601) &**
	Ring Theory and Linear Algebra – II (UPC- 32351602)
Name of the Course	: **B.Sc. (H) Mathematics**
Semester	: **VI Semester**
Duration	: **2 hours**
Maximum Marks	: **75 Marks**

Instructions for Candidates :

Do any Four questions. All questions carry equal marks.

1. Use Cauchy Riemann Equations to prove that if f is analytic in a domain D and $f'(z)$ is constant, then $f(z)$ must be a complex linear function. Also, if $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, then what can be said about f throughout D ?

2. Is an entire function constant ? Provide a justification of your answer with help of proof or a suitable example ?

3. What can you say about the integral of any analytic function f defined on a domain D over any contour lying entirely in D. Discuss all cases over any contour in D and for any analytic function f.

4. Prove or disprove that the given function is an inner product on vector space R^3

$$\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1 y_1 + x_3 y_3.$$

Let V be a vector space over F. Prove or disprove that sum of two inner products on V is also an inner product on V. Is the difference of two inner products on V an inner product over V ? Justify your answer with a suitable example.

Let W be a subspace of R^4 consisting of all vectors which are orthogonal to the vectors $u = (1,\ 0, -1, 1)$ and $v = (2, 3, -1, 2)$. Find a basis for W. Further, prove or disprove that $R^4 = W \oplus W^{\perp}$.

5. For an inner product space V, given vectors $x, y \in V$ with the property that $|\langle x, y \rangle| = \|x\| \|y\|$ then prove or disprove that x and y are independent.

Verify whether the operator $T_1(x, y) = (y, -x)$ defined on R^2 is self-adjoint. Given a self-adjoint operator T defined on a real inner product space V, such that $\langle T(u), u \rangle = 0$ for every $u \in V$ then prove that $T = T_0$. Is the same conclusion valid for T_1 defined on R^2 ? Further discuss if the result is necessarily true if T is any linear operator defined on a complex inner product space.

Prove or disprove that every self-adjoint operator is normal. Discuss if the converse is true or not, in general via a suitable example. Hence or otherwise, provide a proof of the converse statement by specifying the conditions on the underlying inner product space V and the characteristic polynomial of the operator T under consideration.

6. Find the sum and product of the polynomials $f(x) = 4x - 5$, $g(x) = 2x^2 - 4x + 2$ in $Z_8[x]$. Prove or disprove that $Z[\sqrt{-6}]$ is not a UFD. Verify whether the polynomial $f(x) = x^3 + x^2 - 2x - 1$ is irreducible over Q.

Let R be an integral domain with unity. Prove or disprove the statement that every irreducible element in $R[x]$ is also an irreducible polynomial. Is the converse true ? Justify your answer via an example.