ResearchGate

See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/282331419

Generalizations of Prime Subsemimodules

Article *in* Southeast Asian Bulletin of Mathematics · January 2015

CITATION
1

READS
135

1 author:

Manish Kant Dubey
Defence Research and Development Organisation
27 PUBLICATIONS **45** CITATIONS

SEE PROFILE

All content following this page was uploaded by Manish Kant Dubey on 01 October 2015.

The user has requested enhancement of the downloaded file.

Southeast Asian Bulletin of Mathematics (2015) 39: 469–476

Southeast Asian
Bulletin of
Mathematics
© SEAMS. 2015

Generalizations of Prime Subsemimodules

Manish Kant Dubey
SAG, Metcalf House, DRDO Complex, Delhi 110054, India
Email: kantmanish@yahoo.com

Poonam Sarohe
Department of Mathematics, Lakshmibai College, University of Delhi, Delhi 110052, India
Email: poonamsarohe@gmail.com

Received 10 July 2013
Accepted 19 May 2014

Communicated by Yuqun Chen

AMS Mathematics Subject Classification(2000): 16Y60

Abstract. Let S be a commutative semiring with unity and M be an unitary left S-semimodule. Let $\phi : T(M) \rightarrow T(M) \cup \{\emptyset\}$ be a function, where $T(M)$ is the set of all subsemimodules of M. A proper subsemimodule N of M is called a ϕ-prime subsemimodule of M if $s \in S$, $x \in M$, $sx \in N \setminus \phi(N)$, then either $s \in (N : M)$ or $x \in N$. In this paper, we study the concept of ϕ-prime subsemimodule which is a generalization of ϕ-prime ideal in a commutative ring and give some characterizations in terms of M-subtractive subsemimodules.
Keywords: Semiring; Semimodule; ϕ-prime subsemimodule; M-subtractive subsemimodule.

1. Introduction

D.D. Anderson and E. Smith in [2], have first introduced the concept of a weakly prime ideal in a commutative ring with unity for the study of factorizations in a commutative ring with zero divisors. S.M. Bhatwadekar and P.K. Sharma in [10], have extended the concept in terms of almost prime ideals in commutative ring with non zero identity. Further, D.D. Anderson and M. Bataineh in [1] have generalized this concept and define the notion of ϕ-prime ideals in commutative ring. Recently, N. Zamani in [16], has introduced the notion of ϕ-prime submodule and prove analogous of some of the properties of ϕ-prime ideals of commutative ring for ϕ-prime submodules. Recent work on the topics of semimodules have been considered and studied in [8], [9], [3], [6], [7], [10] and [13].

In this paper, we have extended the results of prime ideals of commutative ring to prime subsemimodules of semimodules. By a commutative semiring we mean a commutative semigroup $(S, \cdot)$ and a commutative monoid $(S, +, 0_s)$ in which 0_s is the additive identity and $0_s \cdot x = x \cdot 0_s = 0_s$ for all $x \in S$, both are connected by the ring like distributivity. A non-empty subset I of a semiring S is called an ideal of S if $a, b \in I$ and $s \in S$, then $a + b \in I$ and $sa, as \in I$. An ideal I of a semiring S is called subtractive if $a, a + b \in I$, $b \in S$ then $b \in I$. A prime ideal P of a semiring S is an ideal with the property that for $a, b \in S$, $ab \in P$ implies $a \in P$ or $b \in P$. A proper ideal P of a commutative semiring is called weakly prime if $0 \neq ab \in P$ implies $a \in P$ or $b \in P$. A prime ideal is always a weakly prime ideal of a semiring S but converse needs not be true.

Let S be a semiring. A left S-semimodule M is a commutative monoid $(M, +)$ which have a zero element 0_M, together with an operation $S \times M \to M$, denoted by $(a, x) \to ax$ such that for all $a, b \in S$ and $x, y \in M$,

(1) $a(x + y) = ax + ay$,
(2) $(a + b)x = ax + bx$,
(3) $(ab)x = a(bx)$,
(4) $0_s \cdot x = 0_M = a \cdot 0_M$.

A non-empty subset N of a left S-semimodule M is called subtractive if $a, a + b \in N$, $b \in M$ then $b \in N$.

A left S-semimodule M is called cyclic if M can be generated by a single element, that is, $M = \langle m \rangle = Sm = \{sm : s \in S\}$ for some m in M. A left S-semimodule M is called cancellative if whenever $rx = sx$ for elements $r, s \in S$ and $x \in M$ then $r = s$. Let M be a left S-semimodule. An equivalence relation ρ of M is said to be a congruence relation if $(a, b) \in \rho$ implies $(a + c, b + c) \in \rho$ for all $c \in M$ and $(ra, rb) \in \rho$ for all $r \in S$. Let N be a subsemimodule of M. The Bourne relation (Latorre 1965) ρ on M is defined as

$$\rho = \{(x, y) \in M \times M : x + i = y + j \text{ for some } i, j \in N\}.$$

Then ρ is a congruence relation on M. Hence M/ρ can be made into a left S-semimodule under $\oplus$ and $\odot$ defined by

$$x\rho \oplus y\rho = (x + y)\rho \quad \text{and} \quad a \odot x\rho = (ax)\rho\,.$$

This left S-semimodule is called the quotient semimodule of M modulo N and is denoted by M/N. An element $s \in S$ is called a zero-divisor for a semimodule M if $sm = 0$ for some nonzero element m of M. $Z_S(M)$ denotes the set of all zero divisors of M. A proper subsemimodule N of M is said to be a prime subsemimodule of M, if $ax \in N$, $a \in S$ and $x \in M$ then either $x \in N$ or

$a \in (N : M)$ where $(N : M) = \{a \in S : aM \subseteq N\}$ is an associated ideal of S. A proper subsemimodule N of M is said to be a weakly prime if whenever $a \in S$, $x \in M$, $0 \neq ax \in N$ implies $x \in N$ or $a \in (N : M)$. It is easy to prove that prime subsemimodule is always a weakly prime subsemimodule of an S-semimodule M.

Throughout this paper, S will always denote a commutative semiring with identity $1 \neq 0$ and left S-semimodule means unitary semimodules.

2. ϕ-Prime Subsemimodules

In this section we introduce the notion of ϕ-prime subsemimodules of M and give their characterizations.

Definition 2.1. *Let $T(M)$ be the set of all subsemimodules of M and $\phi : T(M) \to T(M) \cup \{\emptyset\}$ be a function. A proper subsemimodule N of M is called a ϕ-prime subsemimodule of M if $s \in S$, $x \in M$ and $sx \in N \backslash \phi(N)$ implies that $s \in (N : M)$ or $x \in N$.*

As $N \setminus \phi(N) = N \setminus (N \cap \phi(N))$, so without loss of generality, we assume throughout the paper that $\phi(N) \subseteq N$. The illustration of ϕ-prime subsemimodule is as follows. Let S be a commutative semiring and $\phi : T(M) \to T(M) \cup \{\emptyset\}$ be a function. Define

$$\begin{aligned}
&\phi_\emptyset(N) = \emptyset, \quad \forall\ N \in T(M)\\
&\phi_0(N) = \{0\}, \quad \forall\ N \in T(M)\\
&\phi_1(N) = (N : M)N, \quad \forall\ N \in T(M)\\
&\phi_2(N) = (N : M)^2 N, \quad \forall\ N \in T(M)\\
&\phi_\omega(N) = \bigcap_{i=1}^{\infty} (N : M)^i N, \quad \forall\ N \in T(M)
\end{aligned}$$

It is clear that $\phi_\emptyset$, ϕ_0-prime subsemimodules are prime, weakly prime subsemimodules respectively.

Definition 2.2. *Let S be a semiring and M be an S-semimodule. Then a proper subsemimodule N of M is called M-subtractive subsemimodule of M, if N and $\phi(N)$ (that is, $\phi_0(N), \phi_1(N), .., \phi_\omega(N)$), both are subtractive subsemimodules of M.*

Clearly, every M-subtractive subsemimodule of M is a subtractive subsemimodule of M and it is easy to see that for any $M-$subtractive subsemimodule and for every positive integer n, we have prime subsemimodules $\Rightarrow$ ϕ_ω-prime $\Rightarrow$ ϕ_n-prime $\Rightarrow$ ϕ_{n-1}-prime.

Example 2.3. Every submodule of an R-module is an M-subtractive.

Example 2.4. Let S be $Z^* = (Z_0^+, +, \cdot)$.

(1) Consider the subsemimodule $N = 2Z_0^+$ of an S-semimodule $M = (Z_0^+, +)$. Then the associated ideal $(N : M)$ of N is $\{0, 2, 4, \ldots\}$. Clearly, $\phi_1(N)$ is a subtractive subsemimodule of M. Therefore N is an M-subtractive subsemimodule of M.

(2) Consider $N = 2Z^* \times 2Z^*$ be a subsemimodule of an S-semimodule $M = Z^* \times Z^*$. Then N and $\phi_1(N)$ are subtractive subsemimodules of M. Therefore, N is an M-subtractive subsemimodule of M.

(3) Consider the semimodule $M = (Z_{12}, +)$, where Z_{12} is the set of all positive integers modulo 12. Then $N = \{0, 4, 8\}$ is a subtractive subsemimodule of M and the associated ideal $(N : M)$ of N is $\{0, 4, 8\}$ and so $\phi_1(N) = \{0, 4, 8\}$. Therefore, $\phi_1(N)$ is subtractive. Consequently, N is an M-subtractive subsemimodule of M.

Result 2.5. *Let M be an S-semimodule and N be a proper subsemimodule of M. Then for $x \in M$, the following holds.*

(1) *If N is subtractive then $(N : M)$ is a subtractive ideal of S.*

(2) *If N is subtractive then $(N : x)$ is a subtractive ideal of S, where $(N : x) = \{r \in S : rx \in N\}$.*

(3) *If N is subtractive then $(0 : M)$ is a subtractive ideal of S, where $(0 : M) = \{r \in S : rM = 0\}$.*

Proof. Proofs are elementary and hence omitted. ∎

Result 2.6. [13] *Let I and J be two subtractive ideals in S. Then $I \cup J$ is a subtractive ideal of S if and only if $I \cup J = I$ or $I \cup J = J$.*

Proof. Proof is straight forward. ∎

Theorem 2.7. *Let S be a commutative semiring and M be an S-semimodule. Let $\phi : T(M) \to T(M) \cup \{\emptyset\}$ be a function and N be a ϕ-prime M-subtractive subsemimodule of M such that $(N : M)N \nsubseteq \phi(N)$. Then N is a prime subsemimodule of M.*

Proof. Let N be a ϕ-prime M-subtractive subsemimodule of M and $ax \in N$ for some $a \in S$ and $x \in M$. Let $ax \notin \phi(N)$, then $ax \in N \setminus \phi(N)$, which gives, $a \in (N : M)$ or $x \in N$, as N is a ϕ-prime subsemimodule of M. Therefore, N is prime. So, let $ax \in \phi(N)$. Then we can assume that $aN \subseteq \phi(N)$ because if $aN \nsubseteq \phi(N)$, then there exists $n \in N$ such that $an \notin \phi(N)$ and $an \in N$. Therefore, $a(x+n) \in N \setminus \phi(N)$. Thus we have either $a \in (N : M)$ or $x + n \in N$, that is, $a \in (N : M)$ or $x \in N$, as N is an M-subtractive subsemimodule of M. So N is a prime subsemimodule of M. Next, suppose that $(N : M)x \subseteq \phi(N)$.

If $(N : M)x \nsubseteq \phi(N)$, then there exists $u \in (N : M)$ such that $ux \in (N : M)x$ but $ux \notin \phi(N)$. This implies, $(a+u)x \in N \setminus \phi(N)$. Since N is a ϕ-prime M-subtractive subsemimodule, we have either $a+u \in (N : M)$ or $x \in N$, that is, $a \in (N : M)$ or $x \in N$. Therefore, N is prime. Since $(N : M)N \nsubseteq \phi(N)$, then there exist some $r \in (N : M)$ and $n_1 \in N$ such that $rn_1 \notin \phi(N)$. So $(a+r)(x+n_1) \in N \setminus \phi(N)$ and hence $(a+r) \in (N : M)$ or $(x+n_1) \in N$, that is, $a \in (N : M)$ or $x \in N$. Therefore, in any case, we have N is a prime subsemimodule of M. ∎

Corollary 2.8. *Let N be a weakly prime M-subtractive subsemimodule of an S-semimodule M such that $(N : M)N \neq \{0\}$. Then N is a prime subsemimodule of M.*

Proof. The proof is obvious by taking $\phi = \phi_0$ in the above theorem. ∎

Corollary 2.9. *Let M be an S-semimodule and N be a ϕ-prime M-subtractive subsemimodule of M such that $\phi(N) \subseteq (N : M)^2N$. Then for every $a \in S$ and $x \in M$, $ax \in N \setminus \bigcap_{i=1}^{\infty}(N : M)^iN$ implies that $a \in (N : M)$ or $x \in N$, that is, N is a ϕ_ω-prime.*

Proof. If N is a prime subsemimodule of M, then there is nothing to prove. Suppose N is not a prime subsemimodule of M. Then by above theorem, we have $(N : M)N \subseteq \phi(N) \subseteq (N : M)^2N \subseteq (N : M)N$. This implies $\phi(N) = (N : M)N = (N : M)^2N$. Thus, $\phi(N) = (N : M)^iN$ for all $i \geq 1$. Hence, N is a ϕ_ω-prime subsemimodule of M. ∎

Theorem 2.10. *Let M be a cancellative S-semimodule and $0 \neq x \in M$ be such that $Sx \neq M$ and Sx is an M-subtractive subsemimodule of M. If Sx is not a prime subsemimodule of M, then Sx is not ϕ_1-prime subsemimodule of M.*

Proof. Since Sx is not a prime subsemimodule of M, then there exist $a \in S$ and $y \in M$ such that $a \notin (Sx : M)$ and $y \notin Sx$, but $ay \in Sx$. Suppose Sx is a ϕ_1-prime subsemimodule of M and let $ay \notin (Sx : M)Sx$. Then $ay \in Sx \setminus (Sx : M)Sx$ gives $a \in (Sx : M)$ or $y \in Sx$, a contradiction.

Again, let $ay \in (Sx : M)Sx$. We have $y + x \notin Sx$ (as Sx is subtractive) and $a(y+x) \in Sx$. If $a(y+x) \notin (Sx : M)Sx$, then $a(y+x) \in Sx \setminus (Sx : M)Sx$ gives $a \in (Sx : M)$ or $y+x \in Sx$, that is, $a \in (Sx : M)$ or $y \in Sx$, a contradiction. So, we let $a(y+x) \in (Sx : M)Sx$. Then $ax \in (Sx : M)Sx$ (as Sx is M-subtractive). Therefore, $ax = r \cdot 1 \cdot x = rx$, for some $r \in (Sx : M)$ and M is an unitary semimodule, which gives $a = r$, as M is a cancellative semimodule. Therefore, we have $r = a \in (Sx : M)$, again we get a contradiction. Hence, Sx is not a ϕ_1-prime subsemimodule of M. ∎

Corollary 2.11. *Let M be a cancellative S-semimodule and $0 \neq x \in M$ be such that $Sx \neq M$ and Sx is an M-subtractive subsemimodule of M. Then Sx is a* *prime subsemimodule of M if and only if Sx is a ϕ_1-prime subsemimodule of M.*

Definition 2.12. *Let M be an $S-$semimodule. Then a semimodule M is called $M-$cancellative if whenever $rm = rn$ for elements $m, n \in M$ and $r \in S$ then $m = n$.*

Further, we give another characterization of ϕ_1-prime subsemimodule of M.

Theorem 2.13. *Let M be an M-cancellative S-semimodule and $a \in S$ be such that $aM \neq M$. Let aM be an M-subtractive subsemimodule of M. Then aM is a ϕ_1-prime subsemimodule of M if and only if aM is a prime subsemimodule of M.*

Proof. Suppose aM is a ϕ_1-prime subsemimodule of M. Let $rx \in aM$, where $r \in S$ and $x \in M$. If $rx \notin (aM : M)aM$, then $r \in (aM : M)$ or $x \in aM$, as aM is a ϕ_1-prime subsemimodule of M. Therefore, aM is a prime subsemimodule of M. So, we can assume $rx \in (aM : M)aM$. Also, $(r + a)x \in aM$. If $(r + a)x \notin (aM : M)aM$, then $(r+a)x \in aM \backslash (aM : M)aM$, this implies $(r+a) \in (aM : M)$ or $x \in aM$, that is, $r \in (aM : M)$ or $x \in aM$. Hence, the result follows. Again, suppose $(r + a)x \in (aM : M)aM$, which gives $ax \in (aM : M)aM$, as aM is an M-subtractive. Therefore, there exists $y \in (aM : M)M$ such that $ax = ay$, which gives $x = y$ (as M is an M-cancellative). Hence, $x = y \in (aM : M)M \subseteq aM$. Consequently, aM is a prime subsemimodule of M. Converse is obvious. ■

Theorem 2.14. *Let S be a semiring and M be an S-semimodule. Let N be a proper $M-$subtractive subsemimodule of M. Then the following statements are equivalent:*

(1) *N is a ϕ-prime subsemimodule of M;*
(2) *If $x \in M \setminus N$, then $(N : x) = (N : M) \cup (\phi(N) : x)$;*
(3) *If $x \in M \setminus N$, then $(N : x) = (N : M)$ or $(N : x) = (\phi(N) : x)$.*

Proof. (1)⇒(2) Let $x \in M \setminus N$ and $r \in (N : x)$. Then $rx \in N$. If $rx \in \phi(N)$, then $r \in (\phi(N) : x)$. Therefore, the result follows. Again, if $rx \notin \phi(N)$, then $rx \in N \setminus \phi(N)$, therefore, $r \in (N : M)$, because N is a ϕ-prime subsemimodule of M. Thus, we have $(N : x) = (N : M) \cup (\phi(N) : x)$.

(2)⇒(3) Let $(N : x) = (N : M) \cup (\phi(N) : x)$ for $x \in M \setminus N$. Then either $(N : x) = (N : M)$ or $(N : x) = (\phi(N) : x)$ because if an ideal is a union of two subtractive ideals then it is equal to one of them (by Result 2.6). Therefore, the inclusion follows.

(3)⇒(1) Suppose that $rx \in N \setminus \phi(N)$ for some $r \in S$ and $x \in M$. Then $rx \in N$ and $rx \notin \phi(N)$ implies $r \in (N : x)$ and $r \notin (\phi(N) : x)$. Therefore, $(N : x) \neq (\phi(N) : x)$. Hence, by given assumption, we have $(N : x) = (N : M)$. Therefore, $r \in (N : M)$ and hence N is a ϕ-prime subsemimodule of M. ■

Corollary 2.15. *Let S be a semiring and M be an S-semimodule. Let N be a proper subtractive subsemimodule of M. Then the following statements are equivalent*:

(1) *N is a weakly prime subsemimodule of M.*
(2) *$(N : x) = (N : M) \cup (0 : x)$, for any $x \in M \setminus N$.*
(3) *$(N : x) = (N : M)$ or $(N : x) = (0 : x)$, for any $x \in M \setminus N$.*

Proof. The proof is follows from above theorem by taking $\phi = \phi_0$. ■

Theorem 2.16. *Let N be a proper M-subtractive subsemimodule of M. Then N is a ϕ-prime subsemimodule of M if and only if* IP $\subseteq N \setminus \phi(N)$ *for some ideal I of S and a subsemimodule P of M, implies either $I \subseteq (N : M)$ or $P \subseteq N$.*

Proof. Suppose N is a ϕ-prime subsemimodule of M. Let I be an ideal of S and P be a subsemimodule of M such that $IP \subseteq N \setminus \phi(N)$. Suppose $P \nsubseteq N$. We show $I \subseteq (N : M)$. Let $a \in I$ and $x \in P \setminus N$. Then $ax \in IP \subseteq N \setminus \phi(N)$. By Theorem 2.14, $(N : x) = (N : M)$ or $(N : x) = (\phi(N) : x)$. Since $ax \in N \setminus \phi(N)$, therefore $(N : x) \neq (\phi(N) : x)$. Hence $(N : x) = (N : M)$. Thus $a \in (N : M)$. This implies that $I \subseteq (N : M)$. Conversely, let $ax \in N \setminus \phi(N)$ for some $a \in S$ and $x \in M$. Considering the ideal generated by a, $\langle a\rangle$ and subsemimodule generated by x, $\langle x\rangle$, we have $\langle a\rangle\langle x\rangle \subseteq N \setminus \phi(N)$. By given supposition, we have $\langle a\rangle \subseteq (N : M)$ or $\langle x\rangle \subseteq N$ and thus $a \in (N : M)$ or $x \in N$. Hence, N is a ϕ-prime subsemimodule of M. ■

Proposition 2.17. *Let N be a ϕ_1-prime M-subtractive subsemimodule of M. Then the following holds:*

(1) *If a is a zero divisor in M/N, then $aN \subseteq (N : M)N$.*
(2) *Let I be an ideal of S such that $(N : M) \subseteq I$ and $I \subseteq Z_S(M/N)$. Then $IN = (N : M)N$.*

Proof. (1) Since a is a zero divisor in M/N, therefore there exists $x \in M\setminus N$ such that $ax \in N$. If $a \in (N : M)$, then clearly $aN \subseteq (N : M)N$. So let $a \notin (N : M)$. Then we must have $ax \in (N : M)N$, as N is a ϕ_1-prime subsemimodule of M. Let $y \in N$. Then $y + x \notin N$ and $a(y + x) \in N$ (as N is subtractive). Since N is a ϕ_1-prime subsemimodule of M, therefore $a(y+x) \in (N : M)N$, which gives $ay \in (N : M)N$. Hence $aN \subseteq (N : M)N$, which is required.

(2) The result is follows from (1). ■

Theorem 2.18. *Let S be a commutative semiring and M be a cyclic S-semimodule. Let N be a ϕ_1-prime subsemimodule of M. Then $(N : M)$ is a ϕ_2-prime ideal of S.*

Proof. Let $ab \in (N : M) \setminus \phi_2(N : M)$ for some $a, b \in S$ and $a \notin (N : M)$. Let $M = Sx$. Therefore, $ab \in (N : M)$ and $ab \notin \phi_2(N : M)$. We have

$(N:M)^2 = ((N:M)N:M)$. This gives $abM \subseteq N$ and $abM \nsubseteq (N:M)N$. So, $abx \in N$ but $abx \notin (N:M)N$, that is, $abx \in N \setminus \phi_2(N)$. Therefore $b \in (N:M)$, because $ax \notin N$ and N is a ϕ_1-prime subsemimodule of M. Hence $(N:M)$ is a ϕ_2-prime ideal of S. ■

References

[1] D.D. Anderson and M. Bataineh, Generalizations of prime ideals, *Comm. Algebra* **36** (2008) 686–696.

[2] D.D. Anderson and E. Smith, Weakly prime ideals, *Houston J. Math.* **29** (2003) 831–840.

[3] M.R. Adhikari and P. Mukhopadhyay, Exact sequences of semimodules, *Bull. Cal. Math. Soc.* **94** (1) (2002) 23–32.

[4] P.J. Allen, N. Eggers, H.S. Kim, Ideal theory in commutative A-semiring, *Kyungpook Math. Journal* **26** (2006) 261–271.

[5] S.E. Atani, On primal and weakly primal ideals over commutative semirings, *Glas. Math.* **43** (2008) 13–23.

[6] R.E. Atani and S.E. Atani, Spectra of semimodules, *Bull. Acad. Stiiunte. Rep. Mold. Matematica* **3** (2011) 15–28.

[7] R.E. Atani and S.E. Atani, On subsemimodules of semimodules, *Bull. Acad. Stiiunte. Rep. Mold. Matematica* **2** (2010) 20–30.

[8] S.K. Bhambri and M.K. Dubey, Extensions of semimodules and injective semimodules, *Southeast Asian Bull. Math.* **34** (1) (2010) 25–41.

[9] S.K. Bhambri, M.K. Dubey, Anuradha, On prime, weakly prime left ideals and weakly regular ternary semirings, *Southeast Asian Bull. Math.* **37** (6) (2013) 801–811.

[10] S.M. Bhatwadekar and P.K. Sharma, Unique factorization and birth of almost primes, *Comm. Algebra* **33** (2005) 43–49.

[11] J. Dauns, Prime modules, *J. Reine Angew. Math.* **298** (1978) 156–181.

[12] J.S. Golan, The theory of semirings with applications in mathematics and theoretical computer science, In: *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Longman Scientific and Technical, Harlow UK, 1992.

[13] V. Gupta and J.N. Chaudhari, Characterization of weakly prime subtractive ideals in semirings, *Bull. Inst. Math. Acad. Sin. (N.S.)* **3** (2) (2008) 347–352.

[14] D.R. Latore, On h-ideals and k-ideals in Hemiring, *Bull. Math. Debrecen* **12** (1965) 219–226.

[15] C.P. Lu, Prime submodules, *Comm. Math. Univ. Sancti. Pauli* **33** (1984) 61–69.

[16] N. Zamani, ϕ-prime submodules, *Glasgow Math. Journal* **52** (2010) 253–259.

View publication stats